Tracking of Fading Channels Using Combination of NLMS and RLS

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***Abstract*:** Adaptive filters combination approach provides the diversity that can be used to amplify performance beyond the ability of individual adaptive filter. This paper presents the tracking behavior of the combination of Normalized Least Mean Square (NLMS) algorithm and the Recursive Least Square (RLS) algorithm in nonstationary environment modeled as time-varying Rayleigh sequences arising due to different Doppler shifts. The main performance metrics to evaluate the techniques are the mean squared error and mean squared deviation. Simulation results validate the superiority of combining NLMS and RLS over component algorithms, regarding fast convergence and reduced steady state MSE.

Introduction:

The algorithm's capacity to follow the statistical variations of the signal is one of the significant focuses to be viewed as, while adopting an appropriate algorithm for a given application. These variations in channels result from Doppler's shift in a high velocity scenario, like fast moving objects. The degree of Doppler shifts produced depends on circumstances; such as, the transmitting frequency, slope of terrain, obstructions and relative moment of the transmitter and receiver[1], [2]. The issue is aggravated by multipath impacts because of reflections, dispersing and diffraction, causing inter-symbol interference (ISI)[3]. Although the gradient based-LMS algorithm ,performs better in steady state, it suffers from a slow convergence rate[4], [5]. Faster convergence is one of the preeminent property of RLS but degradation of its steady-state behavior in non-stationary environment is its drawback [6]. The ability of LMS and RLS for tracking is influenced by the surrounding environment [7]–[9]Variable step size adaptive schemes can be a viable solution for achieving fast convergence as well as lower steady state error. However it is demonstrated by [10] that there is no single VS-LMS algorithm that is perfect for all applications. In a variety of contexts, the issue of channel tracking has been studied. A few related studies are mentioned here. Capability of decision directed (DD) maximum likelihood (ML) channel tracking algorithm is analyzed in [11] . Kalman tracking is utilized for performance evaluation of MIMO-OFDM communication ,and low mobility channel is considered with inter-block fluctuation[12] . A mathematical approach, the "reward –punishment" rule is utilized to get a variable step size algorithm ,and then applied to estimate the nonstationary Rayleigh fading channel [13]. Combination of GVSS-LMS and MKF algorithms [14] and frame splitting [15] is used for OFDM . For massive MIMO systems, channel tracking is inspected in [16].The efficiency of Kalman filter for MIMO is researched [17], scheme for joint CFO with Channel estimation is introduced by [18]. Spline adaptive filters [20] and least squares [21] have been applied for tracking of channels. Frictional-order algorithms are used for tracking of Rayleigh fading channels [22].

The idea to add up the outputs of few different adaptive algorithms for achievement of superior results than that of a component filter was at first introduced by [23] ,which was latter upgraded by [24], [25]. Previous works were mostly confined to combine the adaptive filters of the same families, i.e., two Least Mean Square [26]–[28], two recursive Least Square [26] or two Constant Modulus Algorithm [29] filters with distinct controlling parameters, such as μ or λ. Combination of different algorithms to get improved performance was proposed by [27] and it was also proved that the combination is universal and can be applied to distinct option of algorithms. Combination of

different algorithms have been utilized in a diversity of applications such as adaptive line enhancement [30], characterization of signal modality[31], for sparse or acoustic echo cancelation[32]–[35], array beamforming [36], [37], and active noise control[38], [39]. The combination of adaptive filters endeavors the divide and conquer principle that has also been utilized by the machine-learning community (e.g., in bagging or boosting[40]). This paper presents the combination of NLMS and RLS algorithm for fast convergence and lower steady state error.

The remainder of the paper is organized as follows: In next section, system model and notations are shown. In section 3, tracking phenomenon is presented first, then LMS and RLS algorithms along with their variants ;NLMS and E-RLS are described. Simulation results are presented in section 4.In the end, conclusions are drawn.

Problem formulation and notation

Two phenomena i.e., multipath effect and Doppler shift, in conjunction with each other attenuates communication channels. When there is no distinct dominant path between transmitter and receiver then Rayleigh model is used to anticipate and mitigate the effects of different types of fading accurately[41]. Rayleigh fading channels provides a realistic illustration of time-varying environment. In such case the optimal Weiner Hopf tap weight; $\mathbf{w}_0 = \mathbf{R}^{-1}\mathbf{p}$,is also time varying. In order to track the alterations in the optimal weights an adaptive filter is required in which weight update equation does not depend upon cross correlation or auto-correlation matrices. Convergence rate, computational complexity, steady state error and filter stability are normally considered while selecting a filter algorithm. These characteristics depend upon structure of filter, error metric and adaptation algorithm.

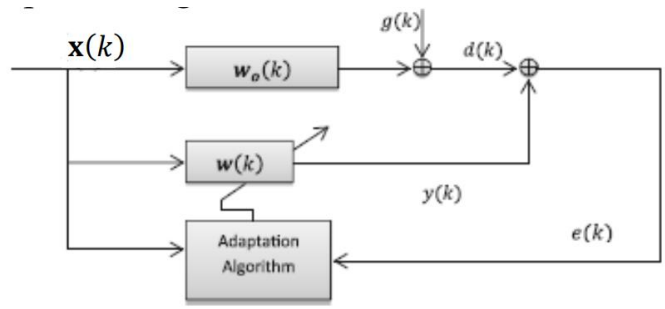

Figure 1: Schematic of adaptive tracking problem

Figure 1 illustrates the configuration of FIR transversal filter with N taps, used for tracking of Rayleigh fading channel .Input data vector $\boldsymbol{x}$ is considered as row vector ,and weight vectors are assumed column vectors. Small letters are used to represent scalars, while capital bold letter denote matrix. k represents the time instant referring to sample number or iteration. We mostly suppose the accessibility of an input vector $\mathbf{x}(k)$ and reference signal $d(k)$,that satisfy the linear regression model ,$d(k)=\mathbf{w}_0(k)\mathbf{x}(k)+g(k)$,by minimization of instantaneous squared error between $d(k)$ and output $y(k)$.Here , $\mathbf{w}_0(k)$ is time-varying optimal Weiner solution and $g(k)$ is a noise sequence, described as a zero mean Gaussian random variable. Output $y(k)$ at time instant k can be described as:

$$y(k)=\sum_{i=0}^{N-1} w_i(k-1)x(k-i) = \mathbf{x}(k)\mathbf{w}(k-1) \quad (1)$$

Where:

$$\mathbf{W}(k) = [w_0(k), w_1(k) \dots, w_{N-1}(k)]^{\mathrm{T}} \quad (2)$$

And

$$\mathbf{x}(k) = [x(k), x(k-1), \dots, x(k-N+1)] \quad (3)$$

Error signal e(n) described as:

$$e(n) = d(n) - y(n) \quad (4)$$

Which can be utilized to generate cost function $\xi(k)$, required to estimate the optimal adaptive filter coefficient. Cost function Gradually reduced to its minimum value, as a result of this operation[42].

During the adaptation process, an adaptive filter begins with an arbitrary initial condition and passes through a transition phase before convergence and reaching its steady state. The performance of an adaptive filter in following changes in the input

signals, after it has arrived at its steady state is termed as *tracking*. A fast convergent algorithm, may or may not be capable of fast tracking [43].

Least Mean Square Algorithm

Minimization of a cost function or particular objective is followed by adaptation of the filter coefficients. The MSE is represented by:

$$\xi = E[e^2(k)] \quad (5)$$
$$= E[|d(k) - y(k)|^2]$$

Replacing $y(k)$ by $\mathbf{w}^{\mathrm{T}}\mathbf{x}(k)$, one obtains:

$$\xi(k) = E[|d(k) - \mathbf{w}^{\mathrm{T}}\mathbf{x}(k)|^2] \quad (6)$$
$$= E[d^2(k)] - 2\mathbf{w}^{\mathrm{T}}\mathrm{E}[d(k)\mathbf{x}(k)] + \mathbf{w}^{\mathrm{T}}E[\mathbf{x}(k)\mathbf{x}^{\mathrm{T}}(k)]\mathbf{w}$$
$$= E[d^2(k)] - 2\mathbf{w}^{\mathrm{T}}\,\mathbf{p} + \mathbf{w}^{\mathrm{T}}\mathbf{R}\mathbf{w}$$

where **R** is the autocorrelation matrix of input signal, and **P** is the cross-correlation vector between the reference signal and the input signal, and can be described as:

$$\mathbf{R} = \mathrm{E}[\mathbf{x}(k)\mathbf{x}^{\mathbf{T}}(k)] \quad (7)$$
$$\mathbf{P} = [\boldsymbol{d}(\mathrm{k})\mathbf{x}^{\mathrm{T}}(k)] \quad (8)$$

Calculating the gradient vector of Equation (6) and equating it to Zero will minimize the MSE cost function and will produce Wiener Solution $\mathbf{w}_0$, given as:

$$\mathbf{w}_0 = \mathbf{R}^{-1}\,\mathbf{p} \quad (9)$$

Inversion of matrix **R** is required to determine the Wiener solution for the MSE calculation, this makes Equation (9) difficult to implement. Wiener solution can be estimated, in a computationally effective manner by using iterative algorithm.

MSE is minimized at each time interval *k*, and filter weights are added with a correction term at time

$k + 1$, that is:

$$\mathbf{w}(k+1) = \mathbf{w}(k) + \Delta\boldsymbol{w}(k) \quad (10)$$

Adaptive filters are designed to handle the formation of this correction term.

When minimum of a function is beyond the realm of analytical solution, then iterative method is used to obtain approximate solution. Steepest descent algorithm utilizes the update rule of equation (10). At each iteration, the algorithm will keep its quest in the direction that will reduce the cost function.

$$\mathbf{w}(k+1) = \mathbf{w}(k) - \frac{\mu}{2}\nabla_{\mathrm{w}}\xi(k) \quad (11)$$

Substituting the gradient of (6), weight update equation becomes:

$$\mathbf{w}(k+1) = \mathbf{w}(k) + \mu[\mathbf{p} - \mathbf{R}\,\mathbf{w}(k)] \quad (12)$$

Equation (12) is the update equation for the steepest descent algorithm. It can be seen that **Error! Reference source not found.** nevertheless requires calculation of **p** and **R**, making it unfeasible for practical implementation. The step-size μ specifies the speed of convergence. Convergence of the steepest-descent mechanism is assured only when $0 < \mu < \frac{1}{\lambda_{max}}$ [43].

The method of the steepest descent is not an adaptive filter but serves as a basis for the LMS algorithm.

Application of instantaneous square error is a simple way to estimate the MSE function, given as $\hat{\xi}(k) = e^2(k)$. Its first order derivative yields:

$$\nabla_w\hat{\xi}(k) = -2e(k)\,\mathbf{x}(k) \quad (13)$$

Substituting this value in (11), will provide LMS weight update equation:

$$\mathbf{w}(k+1) = \mathbf{w}(k) + \mu(k)e(k)\mathbf{x}(k) \quad (14)$$

Step size μ is the single parameter that defines the speed of convergence and steady state MSE of the algorithm. Convergence properties of LMS algorithm are not eminent [6], [44]. Fast convergence can be achieved, utilizing large step-size but it will result in divergence of equation (14).

Normalized LMS algorithm:

The tap input vector $\mathbf{x}(k)$ effects the correction that is applied to the weights of the filter. Therefore, for large values of k, the LMS algorithm faces amplification of gradient noise, which is managed by Normalized least mean squares (NLMS) algorithm, by normalizing the step size μ with the power of the input[45]. Normalized LMS is used to modify its step size accordingly and try to attain fast convergence. Normalized step size is given as:

$$\mu_k = \frac{\bar{\mu}}{\|\mathbf{x}(k)\|^2 + \mathcal{E}} \quad (15)$$

where $\bar{\mu}$ may be considered as different step size parameter, used to manage convergence rate of the algorithm. $\mathcal{E}$ is a small integer introduced to avoid dealing with expected divisions by zero. Substituting μ_k,the weight update equation of LMS is modified to normalized LMS:

$$\mathbf{w}(k+1) = \mathbf{w}(k) + \frac{\bar{\mu}\, e(k)\mathbf{x}(k)}{\|\mathbf{x}(k)\|^2 + \mathcal{E}} \quad (16)$$

Recursive Least Square (RLS) Algorithm:

Weighted least square error cost function is used in RLS, that do not comprise expectations. As optimization procedure is based on all past data, its stochastic nature is reduced in time. The cost function, can be detailed as:

$$\xi(k) = \sum_{l=0}^{k} \rho(l) e^2(l) = \sum_{l=0}^{k} \lambda^{k-l} e^2(l) \quad (17)$$

In above Equation, $0 < \lambda \leq 1$ shows the exponential weighting factor, exploited to influence the current samples extra weightily compared to the earlier samples. The output error, $e(k)$ is calculated as:

$$e(k) = d(k) - w^{\mathrm{T}}(k)\mathrm{x}(k) \quad (18)$$

The basic derivations of RLS are included in [6], [42], [45] and other references. All the terms are acquired from these references for the weight update equation. The autocorrelation matrix **R** has been detailed essentially as the product of input vectors, and **P**, as the product of input-output cross correlation vector, both scaled by λ are given as:

$$\mathbf{R}(k) = \sum_{I=0}^{k} \lambda^{k-1} x^T(l) x(l) \quad (19)$$

$$\mathbf{P}(k) = \sum_{I=0}^{k} \lambda^{k-1} x_k^T(l) y_k(l) \quad (20)$$

A gain vector known as Kalman gain is intended to reduce the calculations and will be used later in the weight update equation [45], given as:

$$q(k) = \frac{R^{-1}(k-1)X^T(k)}{\lambda + R^{-1}(k-1)X^T(k)} \quad (21)$$

Utilizing Kalman gain $q(k)$, the weight update equation can be rewritten as:

$$w(k+1) = w(k) - q(k)e^*(k) \quad (22)$$

And current auto-correlation matrix can be expressed as:

$$\mathbf{R^{-1}}(k) = \frac{1}{\lambda}[R^{-1}(k-1) + \mathrm{q}(k)X^H(k)R^{-1}(k-1)] \quad (23)$$

Extended Recursive Least Square Algorithm:

E-RLS has been found to be equivalent to refined Kalman filter. Good performance of E-RLS is proved for tracking of Rayleigh fading channel [46].Derivation of recursion are already determined in [5], [9], [17], [45]. Utilizing the forgetting factor λ, the a-posteriori error is defined as:

$$r(k) = \lambda^k x(k) P(k-1) x^*(k) \quad (24)$$

The Kalman gain constant is described as:

$$q(k) = \frac{P(k-1)}{r(k)} \quad (25)$$

The a priori error can be expressed as:

$$e(k) = d(k) - h^T(k-1)x(k) \quad (26)$$

The α-scaled update equation for weights is used to acquire the estimated input:

$$h(k) = \alpha h(k-1) + q(k)e(k-1) \quad (27)$$

Eventually,

$$P(k) = |\alpha|^2 \left[P(k-1) - \frac{P(k-1)x^*(k)x(k)P(k-1)}{\lambda^k + x(k)P(k-1)x^H(k)}\right] + \lambda^k\, sI \quad (28)$$

For weight update Equation, variable *h(k)* is replaced by $w(k)$, the recursion for $P(k)$ is multiplied by λ^{-k} ,and weights are zero initialized . Equations (16) and (17) represents weight update equation and $P(k)$ respectively.

$$w(k) = \alpha w(k-1) - \frac{\lambda^{-1}\alpha P(k-1)x^*(k)}{1+\lambda^{-1}x(k)P(k-1)x^*(k)} [d(k) - x(k)w^T(k-1)] \quad (29)$$

$$P(k) = \lambda^{-1}|\alpha|^2 \left[P(k-1) - \frac{\lambda^{-1}P(k-1)x^*(k)x(k)P(k-1)}{1+\lambda^{-1}x(k)P(k-1)x^*(k)}\right] + sI \quad (30)$$

Where, *S* is a positive scalar with $|\alpha| \leq 1$.

Computational Complexity of Different Algorithms:

Algorithm	Mul/Div	Add/Sub
LMS	2M + 1	2M
NLMS	3M + 2	3M
RLS	$4M^2 + 5M$	$2M^2 + 3M$

Simulations:

Rayleigh fading channels are utilized to analyze the tracking capability of the proposed algorithms. Different Doppler shifts are applied to different channels. Mean squared error (MSE) and mean squared deviation (MSD) performance measures are used in simulations to analyze the convergence rate and steady state mean square error.

The MSE and MSD can be described by the relations:

$$\text{MSE(dB)} = 10\log_{10}\text{E}[e^2(k)] = 10\log_{10}\text{E}\left[\left(d(k) - \boldsymbol{u}(k)\boldsymbol{w}(k-1)\right)^2\right] \quad (31)$$

And,

$$\text{MSD} = 10\log_{10}\text{E}[\|\boldsymbol{w_o}(k) - \boldsymbol{w}(k)\|^2] \quad (32)$$

Simulation results are compared with component filters of proposed algorithm; NLMS, RLS and E-RLS.

Rayleigh fading channels are generated using MATLAB and different Doppler shifts are applied. The output of the filter is subjective to Additive White Gaussian noise *g (k)*. 1000 samples are used in experiment for 500 separate runs. Each run is executed for distinct batch of time-varying weights. Channel is assumed to scattered from 20 different objects. Rayleigh density function is used to draw the observations. For RLS and E-RLS, the weights are initialized to zero for each independent run and after convergence the same optimized weights are utilized by NLMS in usual way. *s* and α utilized by RLS and E-RLS ,can be defined as $s = (1-\alpha^2)I$ and $\alpha = J_0(2\pi f_D T_s)$, Where J_0 is a zero-order Bessel function of the first order and can be described to be:

$$J_0(2\pi f_D T_s) = \frac{1}{\pi}\int_0^{\pi} \cos(2\pi f_D T_s \sin\theta)\, d\theta \quad (33)$$

where $f_D = \frac{v f_c}{c}$, is the Doppler frequency, f_c is carrier frequency and v is speed of receiver. c is the speed of light and has the value , 3×10^8 m/s, and T_s represents the sampling period of the input $x(k)$.

Parameter	Type/Value
Fading	Time varying Rayleigh Fading
Noise	AWGN
Algorithm	NLMS, RLS, E-RLS, Combination of NLMS and RLS
Step size (μ)	0.2, 0.3, 0.4
Doppler's frequency	100 Hz, 500 Hz, 1000 Hz
Sampling frequency	1.25 MHz
Forgetting factor (λ)	0.995

Number of iterations	1000
Number of runs	500
Filter length	5

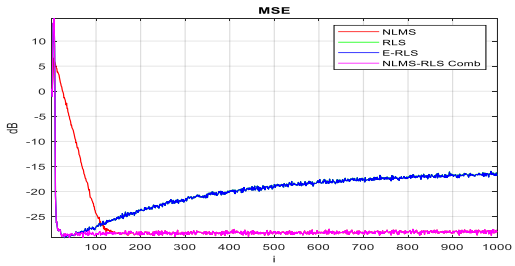

Figure 2: MSE Learning curves for f_D = 100 Hz, μ = 0.2

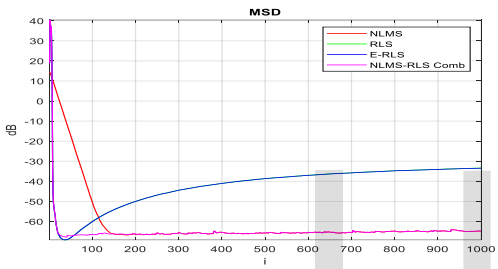

Figure 3: MSD Learning curves for f_D = 100 Hz, μ = 0.2

Figure 2 showed the MSE learning performance for f_D = 100 Hz, μ = 0.2 and λ = 0. 995 with the sampling frequency of 1.25 MHz.RLS and E-RLS has the same convergence point after 11 iterations; at the MSE level of –25 dB. While NLMS algorithm takes 104 iterations to converge. The steady state MSE value for the combination of Normalized Least Square and Recursive Least Square algorithm, E-RLS, NLMS, RLS is [–28.04, –16.68, –28.04, –16.65] dBs respectively.

Figure 3 is the plot of MSD for the above-mentioned same parameters. The steady-state MSD values for the proposed hybrid combination, E-RLS, NLMS, RLS algorithms is [–64.76, –33.75, –64.76, –33.69] dBs, respectively. RLS and E-RLS has identical MSD values.

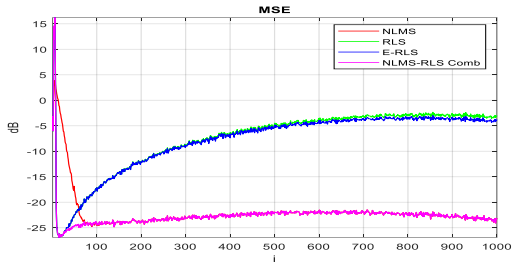

Figure 4: MSE Learning curves for f_D = 500 Hz, μ = 0.3

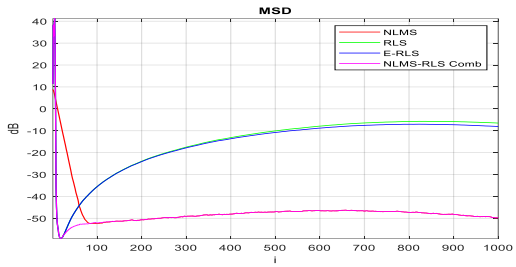

Figure 5: MSD Learning curves for f_D = 500 Hz, μ = 0.3

Figure 4 showed the convergence and MSE curve for f_D = 500 Hz, μ = 0.3 and λ = 0. 995 with the sampling frequency of 1.25 MHz RLS and Extended-RLS converges in 9 iterations with MSE value of –20 db. While the Normalized Least Mean Square converges in 54 iterations. The steady-state MSE values for the combination of RLS and NLMS, E-RLS, N-LMS and RLS are [−23.11, −3.83, −23.11, −3.09] dBs, respectively.

Figure 5 is the plot of MSD for the above mentioned same parameters. The steady-state MSD values for the combination of RLS and NLMS, E-RLS, N-LMS and RLS is [−49.03, −7.64, −49.03, −6.13] dBs, respectively.

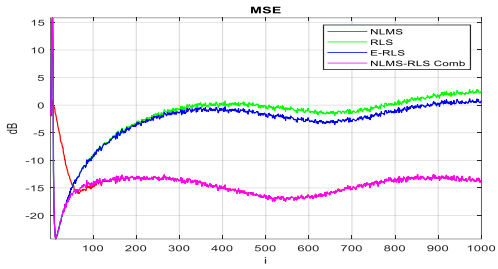

Figure 6: MSE curve for f_D = 1000 Hz and μ =0.2

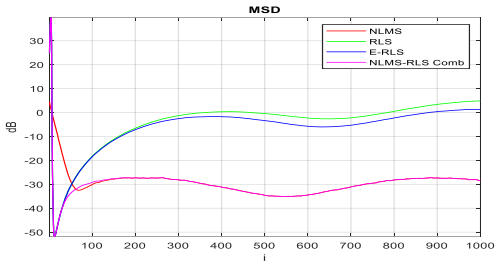

Figure 7: MSD curve for f_D = 1000 Hz and μ =0.2

Figure 6 showed the convergence and MSE curve for f_D = 1000 Hz, μ = 0.2 and λ = 0. 995. RLS and Extended-RLS converges in 8 iterations with -15 dB MSE value. While the Normalized Least Mean Square algorithm achieves this value in 52 iterations. The steady-state MSE for the combination of RLS and NLMS, E-RLS, N-LMS and RLS is [−13.36, 0.53, −13.36, 2.16] dBs, respectively. It can be seen that Recursive Least Square shows the worst behavior at such high frequencies of 1000 Hz.

The MSD performance for the Doppler shift 1000 Hz, λ = 0.995 is shown in Figure 7. The MSD values for combination of NLMS and RLS, E- RLS, NLMS and RLS are [−27.7, 0.9775, −27.72, 4.25] dBs, respectively.

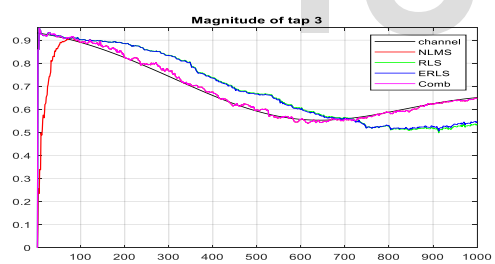

Figure 8: Tracking performance of different algorithms for

tap 3when f_D = 500 Hz, μ=0.3

Figure 8 focuses on tap 3 particularly when f_D = 500 Hz and μ = 0.3. The difference between performance of NLMS, RLS, E-RLS and hybrid configuration to track fast time variation of the Rayleigh channel from start to end clearly appeared in figure, that shows, combination of NLMS and RLS algorithm has a good ability to track fast variation than NLMS and RLS algorithms individually.

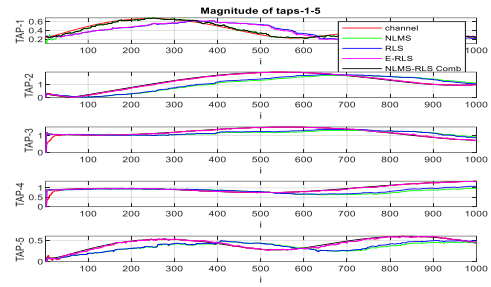

Figure 9: Tracking performance of algorithms for five taps; for

f_D = 1000Hz, μ = 0.4, λ = 0.995

Figure 9 displays the tap weights tracking plots of all algorithm for 5 filter taps sequentially from top to bottom ,for f_D = 1000Hz, μ = 0.4, λ = 0.995, compared with the actual channel to be estimated. The supremacy of combination of RLS-NLMS can be seen in all cases.

Conclusion:

The purpose of the combination of algorithm is to utilize the preeminent properties of component adaptive filters. The input was a time varying Rayleigh fading channel with different Doppler shifts. The proposed algorithm exhibits fast convergence property of RLS and superior steady state performance of NLMS algorithm. At higher Doppler shifts, the proposed algorithm mostly shows better performance in steady state, where the RLS and E-RLS algorithms demonstrate degraded performance. Analyzing all the simulation results show that, combination of adaptive filters approach provides the diversity that can be used to amplify performance beyond the ability of individual adaptive filter.

References

[1] P. Liu, D. W. Matolak, B. Ai, and R. Sun, “Path loss modeling for vehicle-to-vehicle communication on a slope,” *IEEE Trans. Veh. Technol.*, vol. 63, no. 6, 2014.

[2] M. Yang *et al.*, “Path Loss Analysis and Modeling for Vehicle-To-Vehicle Communications with Vehicle Obstructions,” in *2018 10th International Conference on Wireless Communications and Signal Processing, WCSP 2018*, 2018.

[3] K. E. Baddour and N. C. Beaulieu, "Autoregressive modeling for fading channel simulation," *IEEE Trans. Wirel. Commun.*, vol. 4, no. 4, 2005.

[4] J. F. Galdino, E. L. Pinto, and M. S. de Alencar, "Analytical performance of the LMS algorithm on the estimation of wide sense stationary channels," *IEEE Trans. Commun.*, vol. 52, no. 6, 2004.

[5] B. Wittenmark, "Adaptive filter theory. Simon Haykin," *Automatica*, vol. 29, no. 2. 1993.

[6] E. Eweda, "Comparison of RLS, LMS, and Sign Algorithms for Tracking Randomly Time-Varying Channels," *IEEE Trans. Signal Process.*, vol. 42, no. 11, 1994.

[7] P. Sharma and K. Chandra, "Prediction of state transitions in Rayleigh fading channels," *IEEE Trans. Veh. Technol.*, vol. 56, no. 2, 2007.

[8] A. K. Kohli and D. K. Mehra, "Tracking of time-varying channels using two-step LMS-type adaptive algorithm," *IEEE Trans. Signal Process.*, vol. 54, no. 7, 2006.

[9] E. Eleftheriou and D. D. Falconer, "Tracking Properties and Steady-State Performance of RLS Adaptive Filter Algorithms," *IEEE Trans. Acoust.*, vol. 34, no. 5, 1986.

[10] D. Bismor, K. Czyz, and Z. Ogonowski, "Review and comparison of variable step-size LMS algorithms," *Int. J. Acoust. Vib.*, vol. 21, no. 1, 2016.

[11] E. Karami, "Performance analysis of decision directed maximum likelihood MIMO channel tracking algorithm," *Int. J. Commun. Syst.*, vol. 26, no. 12, 2013.

[12] S. Pagadarai, A. M. Wyglinski, and C. R. Anderson, "Low-mobility channel tracking for MIMO-OFDM communication systems," *EURASIP J. Adv. Signal Process.*, vol. 2013, no. 1, 2013.

[13] A. Chatterjee and I. S. Misra, "Design and analysis of reward-punishment based variable step size LMS algorithm in Rayleigh faded channel estimation," in *2015 IEEE Power, Communication and Information Technology Conference, PCITC 2015 - Proceedings*, 2016.

[14] D. S. Kapoor and A. K. Kohli, "Channel estimation and long-range prediction of fast fading channels for adaptive OFDM system," *Int. J. Electron.*, vol. 105, no. 9, 2018.

[15] K. Yonei, K. Maruta, and C. J. Ahn, "Frame splitting and data-aided decision direct channel estimation for OFDM in fast varying fading channels," *ICT Express*, vol. 6, no. 3, 2020.

[16] S. Dahiya and A. K. Singh, "Channel estimation and channel tracking for correlated block-fading channels in massive MIMO systems," *Digit. Commun. Networks*, vol. 4, no. 2, 2018.

[17] C. Komninakis, C. Fragouli, A. H. Sayed, and R. D. Wesel, "Multi-input multi-output fading channel tracking and equalization using Kalman estimation," *IEEE Trans. Signal Process.*, vol. 50, no. 5, pp. 1065–1076, May 2002.

[18] N. H. Cheng, K. C. Huang, Y. F. Chen, and S. M. Tseng, "Maximum likelihood-based adaptive iteration algorithm design for joint CFO and channel estimation in MIMO-OFDM systems," *EURASIP J. Adv. Signal Process.*, vol. 2021, no. 1, 2021.

[19] E. Mostafapour, A. Hoseini, J. Nourinia, and M. C. Amirani, "Channel estimation with adaptive incremental strategy over distributed sensor networks," in *Conference Proceedings of 2015 2nd International Conference on Knowledge-Based Engineering and Innovation, KBEI 2015*, 2016.

[20] M. Scarpiniti, D. Comminiello, G. Scarano, R. Parisi, and A. Uncini, "Steady-state performance of spline adaptive filters," *IEEE Trans. Signal Process.*, vol. 64, no. 4, 2016.

[21] S. Kaddouri, P. P. J. Beaujean, P. J. Bouvet, and G. Real, "Least square and trended doppler estimation in fading channel for high-frequency underwater acoustic

communications," *IEEE J. Ocean. Eng.*, vol. 39, no. 1, 2014.

[22] S. M. Shah, R. Samar, and M. A. Z. Raja, "Fractional-order algorithms for tracking Rayleigh fading channels," *Nonlinear Dyn.*, vol. 92, no. 3, 2018.

[23] P. Andersson, "Adaptive forgetting in recursive identification through multiple models," *Int. J. Control*, vol. 42, no. 5, 1985.

[24] M. NiedźWiecki, "Identification of Nonstationary Stochastic Systems Using Parallel Estimation Schemes," *IEEE Trans. Automat. Contr.*, vol. 35, no. 3, 1990.

[25] M. Niedzwiecki, "Multiple-Model Approach to Finite Memory Adaptive Filtering," *IEEE Trans. Signal Process.*, vol. 40, no. 2, 1992.

[26] J. Arenas-García, M. Martínez-Ramón, Á. Navia-Vázquez, and A. R. Figueiras-Vidal, "Plant identification via adaptive combination of transversal filters," *Signal Processing*, vol. 86, no. 9, 2006.

[27] J. Arenas-García, A. R. Figueiras-Vidal, and A. H. Sayed, "Mean-square performance of a convex combination of two adaptive filters," *IEEE Trans. Signal Process.*, vol. 54, no. 3, pp. 1078–1090, 2006.

[28] J. Arenas-García, A. R. Figueiras-Vidal, and A. H. Sayed, "Tracking properties of a convex combination of two adaptive filters," in *IEEE Workshop on Statistical Signal Processing Proceedings*, 2005, vol. 2005.

[29] J. Arenas-García and A. R. Figueiras-Vidal, "Improved blind equalization via adaptive combination of constant modulus algorithms," in *ICASSP, IEEE International Conference on Acoustics, Speech and Signal Processing - Proceedings*, 2006, vol. 3.

[30] T. Trump, "A combination of two NLMS filters in an adaptive line enhancer," in *17th DSP 2011 International Conference on Digital Signal Processing, Proceedings*, 2011.

[31] B. Jelfs, S. Javidi, P. Vayanos, and D. Mandic, "Characterisation of signal modality: Exploiting signal nonlinearity in machine learning and signal processing," in *Journal of Signal Processing Systems*, 2010, vol. 61, no. 1.

[32] J. Arenas-Garcia and A. R. Figueiras-Vidal, "Adaptive combination of proportionate filters for sparse echo cancellation," *IEEE Trans. Audio, Speech Lang. Process.*, vol. 17, no. 6, 2009.

[33] J. Ni and F. Li, "Adaptive combination of subband adaptive filters for acoustic echo cancellation," *IEEE Trans. Consum. Electron.*, vol. 56, no. 3, 2010.

[34] L. A. Azpicueta-Ruiz, M. Zeller, A. R. Figueiras-Vidal, J. Arenas-Garcia, and W. Kellermann, "Adaptive combination of volterra kernels and its application to nonlinear acoustic echo cancellation," *IEEE Trans. Audio, Speech Lang. Process.*, vol. 19, no. 1, 2011.

[35] L. A. Azpicueta-Ruiz, M. Zeller, A. R. Figueiras-Vidal, W. Kellermann, and J. Arenas-Garcia, "Enhanced adaptive volterra filtering by automatic attenuation of memory regions and its application to acoustic echo cancellation," *IEEE Trans. Signal Process.*, vol. 61, no. 11, 2013.

[36] S. Lu, J. Sun, G. Wang, and J. Tian, "A novel GSC beamformer using a combination of two adaptive filters for smart antenna array," *IEEE Antennas Wirel. Propag. Lett.*, vol. 11, 2012.

[37] D. Comminiello, M. Scarpiniti, R. Parisi, and A. Uncini, "Combined adaptive beamforming schemes for nonstationary interfering noise reduction," *Signal Processing*, vol. 93, no. 12, 2013.

[38] M. Ferrer, A. Gonzalez, M. De Diego, and G. Pinero, "Convex combination filtered-X algorithms for active noise control systems," *IEEE Trans. Audio, Speech Lang. Process.*, vol. 21, no. 1, 2013.

[39] N. V. George and A. Gonzalez, "Convex combination of nonlinear adaptive filters for active noise control," *Appl. Acoust.*, vol. 76, 2014.

[40] L. I. Kuncheva, *Combining Pattern Classifiers*. 2014.

[41] E. S. S. Suthersan, B. Raton, and C. R. C. Press, "Sklar, B. 'Rayleigh Fading Channels,'" *Mob. Commun. Handb.*, pp. 1–40, 1999.

[42] P. S. R. Diniz, *Adaptive filtering: Algorithms and practical implementation*, vol. 9781461441069. 2013.

[43] B. Farhang-Boroujeny, *Adaptive Filters: Theory and Applications, Second Edition*. 2013.

[44] J. Dhiman, S. Ahmad, and K. Gulia, "Comparison between Adaptive filter Algorithms (LMS, NLMS and RLS)," *Int. J. Sci. Eng. Technol. Res.*, vol. 2, no. 5, pp. 2278–7798, 2013.

[45] A. H. Sayed, *Adaptive Filters*, 1st ed. Hobeken: John Wiley & Sons, 2007.

[46] A. S. Mahdi, "A Comparison Between Recursive Least-Squares (RLS) and Extended Recursive Least-Squares (E-RLS) for Tracking Multiple Fast Time Variation Rayleigh Fading Channel," *Al-Khwarizmi Eng. J.*, vol. 12, no. 1, pp. 73–78, 2016.